CHAPTER 2

HYDRAULIC THEORY

Section I. Introduction

2-1. General. This section presents hydraulic design theory, available experimental data and coefficients, and discussions of analysis and problems related to spillway design. Generally, the presentations assume that the design engineer is acquainted with the hydraulic theories involved in uniform flow, gradually and rapidly varied flow, steady and unsteady flow, energy and momentum principles, and other aspects such as energy losses, cavitation, etc., related to hydraulic design. These matters are normally covered in hydraulic handbooks and texts such as those by King and Brater (item 24), Rouse (items 49 and 50), and Chow (item 10). This manual is presented as guidance in the application of textbook material and as additional information not readily available in general reference material. The application of the theory of flow through spillways is based largely upon empirical coefficients, so the designer should deal with maximum and minimum values as well as averages, depending upon the design objective. To be conservative, the designer should generally use maximum loss factors in computing discharge capacity, and minimum loss factors in computing velocities for the design of energy dissipators. As more model and prototype data become available, the range between maximum and minimum coefficients used in design should be narrowed. An example in which the hydraulic design procedures and guidance discussed in this manual are applied to the computation required to design a typical reservoir spillway is shown in Appendix D.

2-2. Basic Considerations. A spillway is sized to provide the required capacity, usually the entire spillway design flood, at a specific reservoir elevation. This elevation is normally at the maximum operating level or at a surcharge elevation greater than the maximum operating level. Hydraulic analysis of a spillway usually involves four conditions of flow:

a. Subcritical flow in the spillway approach, initially at a low velocity, accelerating, however, as it approaches the crest.

b. Critical flow as the water passes over the spillway crest.

c. Supercritical flow in the chute below the crest.

d. Transitional flow at or near the terminus of the chute where the flow must transition back to subcritical.

When a relatively large storage capacity can be obtained above the normal maximum reservoir elevation by increasing the dam height, a portion of the flood volume can be stored in this reservoir surcharge space and the size of the spillway can be reduced. The use of a surcharge pool for passing the spillway design flood involves an economic analysis that considers the added cost of a dam height compared to the cost of a wider and/or deeper spillway. When a gated spillway is considered, the added cost of higher and/or additional gates and piers must be compared to the cost of additional dam height.

When an ungated spillway is considered, the cost of reduced flood-control benefits due to a reduction in reservoir storage must be compared to the cost of additional dam height. The transition of flow from supercritical on the chute to subcritical usually involves considerable energy dissipation. Dissipation of hydraulic energy is accomplished by various methods such as the hydraulic jump, impact, dispersion, etc. The type of energy dissipator used is dependent upon factors that include site geology, the type of dam structure, and the magnitude of the energy to be dissipated. The design discharge for effective energy dissipation is frequently set at the standard project flood rate; however, each facility must be evaluated, and the design discharge used should be dependent upon the damage consequences when the design discharge is exceeded.

Section II. Spillway Discharge

2-3. General.

a. The ogee crest spillway is basically a sharp-crested weir with the space below the lower nappe filled with concrete. The discharge over a spillway crest is limited by the same parameters as the weir, and determined by the following:

$$Q = CL_eH_e^{1.5} \tag{2-1}$$

where

Q = rate of discharge, cubic feet per second (ft^3/sec)
C = coefficient of discharge
L_e = effective length of the crest, feet
H_e = total specific energy above the crest, feet

Extensive investigations of spillway crest shapes, pressures, and coefficients have provided empirical data that will allow the designer to develop a spillway that minimizes the structural size required for the design discharge. Minimization of the structure size is achieved by underdesigning the spillway crest within limits discussed in Chapter 3.

b. An underdesigned crest is defined when the following relationship is true:

$$\frac{H_e}{H_d} > 1$$

where H_d is the crest design head, feet. The design head is a major parameter of the ogee crest shape equation and is discussed in Section II of Chapter 3. Underdesigning the crest results in increasing the discharge coefficient significantly above that of the sharp-crested weir; however, the underdesigned crest results in a reduction of the hydrodynamic pressures on the crest surface. Depending on the degree of underdesigning, the crest pressures can be significantly less than atmospheric.

2-4. Abutment and Piers. All spillways include abutments of some type, and many include intermediate piers. The effect that the abutments and piers have on the discharge must be accounted for; this is accomplished by modifying the crest length using the following equation to determine the effective crest length L_e :

$$L_e = L - 2(nK_p + K_a)H_e \tag{2-2}$$

where

L = net length of crest
n = number of piers
K_p = pier contraction coefficient
K_a = abutment contraction coefficient

2-5. Effect of Approach Flow. Another factor influencing the discharge coefficient of a spillway crest is the depth in the approach channel relative to the design head defined as the ratio P/H_d , where P equals the crest elevation minus the approach channel invert elevation. As the depth of the approach channel P decreases relative to the design head, the effect of approach velocity becomes more significant; and at $P/H_a \leq 1.0$, this effect should not be neglected. The slope of the upstream spillway face also influences the coefficient of discharge. As an example, for $P/H_d > 1$, the flatter upstream face slopes tend to produce an increase in the discharge coefficient. Several investigators have provided data on approach depth and spillway face slope effects. The most recent work has been done by WES (item 28). Data from this work have been used extensively in this manual. The planform of the approach channel can significantly influence the spillway discharge characteristics. The influence of the planform can be evaluated thoroughly only by the use of a site-specific physical model. In some cases a two-dimensional numerical model will be entirely adequate. In the case of a simple spillway approach, analysis of the water surface profile by a standard step method would be sufficient. Spillway approach channels and slope of the upstream spillway face are further discussed in Chapter 3.

Section III. Gradients

2-6. General. The basic principle used to analyze steady incompressible flow through a spillway is the law of conservation of energy expressed by the Bernoulli (energy) equation. The energy equation, generalized to apply to the entire cross section of flow, expresses the energy at any point on the cross section in feet of water by equation 2-3:

$$H = Z + \frac{p}{\gamma} + \alpha \frac{V^2}{2g} \tag{2-3}$$

where

H = total energy head in feet of water above the datum plane
Z = height above a datum plane, feet
p = pressure at the point, pounds per square foot (lb/ft^2>

γ = specific weight of water, pounds per cubic foot (lb/ft^3>
α = energy correction coefficient
V = average flow velocity, feet per second (ft/sec)
g = acceleration due to gravity, ft/sec^2

For most practical problems involving regular-shaped channels, the energy correction coefficient may be taken as unity without serious error.

2-7. Hydraulic and Energy Grade Lines. The hydraulic grade line, also referred to as the mean pressure gradient, may be above, below, or at the free water surface. Defining Z as the invert elevation of a point on the chute, then $Z + p/\gamma$ defines the elevation of the hydraulic grade line at that point. The locus of values of $Z + p/\gamma$ along the spillway describes the mean pressure gradient. The mean pressure gradient at any point along the spillway is always lower than the energy grade line by the value of the mean velocity head at that point. The mean pressure gradient is useful in determining pressures acting on the spillway surface and in determining cavitation potential. For most open channel flow the p/γ term can be replaced by $Y_1 \cos \theta$ where Y_1 is the flow depth normal to the channel bottom and θ is the slope of the channel bottom. Therefore, the sum $Z + Y_1 \cos \theta$ will be equal to the elevation of the water surface at the point and the free surface is the hydraulic grade line for all points on the cross section. For this substitution to be valid, the assumption must be made that the pressure distribution at the point must be hydrostatic, a condition that will occur if conditions are such that vertical acceleration of the flow is negligible and the bed slope is mild. A nonhydrostatic pressure distribution will occur whenever the value of $\cos^2 \theta$ departs materially from unity, such as with steep spillway chutes. The departure of the pressure distribution from hydrostatic due to a steep bed slope does not mean the energy equation cannot be used on steep spillway chutes as a design tool. It means that the designer must recognize that the values derived become increasingly inaccurate as the $\cos^2 \theta$ value departs further from unity. This condition describes one of the basic reasons that physical model studies may be required when designing a spillway.

2-8. Mean Spillway Pressure Computation. The mean pressure at any location along a chute is determined using the principle of conservation of energy as expressed by the energy equation. Conservation of energy requires that the energy at one location on the spillway be equal to the energy at any downstream location plus all intervening energy losses. Expressed in equation form and in units of feet of water

$$Z_1 + \frac{P_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + H_L \qquad (2 - 4a)$$

or for the hydraulic assumption

$$Z_1 + Y_1 \cos \theta + \alpha_1 \frac{V_1^2}{2g} = Z_2 + Y_2 \cos \theta + \alpha_2 \frac{V_2^2}{2g} + H_L \qquad (2 - 4b)$$

Information on the determination of energy losses, H_L , associated with flow over spillways is given in appropriate sections of this engineering manual.

Section IV. Spillway Energy Loss

2-9. General. The determination of hydraulic energy loss associated with flow through a spillway system is important to the design of the training walls, piers, and terminal structure. Energy loss is the direct result of three conditions: boundary roughness (friction), turbulence resulting from boundary alignment changes (form loss), and boundary layer development. Sufficient data and procedures are available to make a reasonably accurate determination of the energy loss during development of the turbulent boundary layer and for fully turbulent flow. Form losses are usually minimal for most spillways; however, when the configuration of a spillway is such that form losses outside the range of experience are encountered, model studies are required. Methods and data necessary for spillway energy loss computations are provided in the following paragraphs.

2-10. **Energy Loss for Fully Developed Turbulent Boundary Layer Flow.**

a. General. Methods for determining the energy loss related to boundary roughness (friction) have been developed by various investigators. The most notable and widely used methods are the Darcy-Weisbach equation, the Chezy equation, and the Manning equation. The Darcy-Weisbach equation involves the direct use of a known effective roughness value, k , from which a boundary resistance (friction) coefficient, f , can be derived for use in the energy loss computation. The Darcy-Weisbach equation is applicable to all fully turbulent flow conditions. The Chezy equation is essentially similar to the Darcy-Weisbach equation in that it involves the direct use of a known effective roughness value and is applicable to all flow conditions. The Manning equation, probably the most commonly used, involves use of an empirically derived resistance coefficient, n , and is considered only applicable to fully turbulent flow. Some investigators such as Strickler have attempted to correlate the Manning's n value to a measured effective roughness value; others have equated the Manning equation to the Darcy-Weisbach equation and to the Chezy equation in order to take advantage of the effective roughness parameter used in those equations. These modifications to the Manning equation have all been accomplished in order to establish some degree of confidence for an otherwise empirically derived roughness coefficient.

b. Darcy-Weisbach Equation. The Darcy-Weisbach equation expresses the energy loss due to boundary roughness in terms of a resistance coefficient, f , as:

$$h_f = \frac{fL}{4R}\left(\frac{V^2}{2g}\right) \qquad (2\text{-}5)$$

where h_f is the energy loss due to friction through a length of channel L having an average hydraulic radius R and an average velocity V . The energy loss has a length dimension (ft-lb/lb) and is usually expressed in feet of water. The resistance coefficient, f , is a dimensionless parameter which can be determined for fully turbulent flow conditions by a form of the Colebrook-White equation

$$f = \left[\frac{1}{2\log\left(\frac{13.8R}{k}\right)}\right]^2 \tag{2-6}$$

or by the Strickler-Manning equation

$$f = 0.113\left(\frac{k}{R}\right)^{1/3} \tag{2-7}$$

which may more accurately derive the resistance coefficient for $R/k > 100$. In both equations 2-6 and 2-7, k is the effective roughness value and R is the hydraulic radius. Both equations 2-6 and 2-7 are valid only for fully turbulent flow defined by the relationship:

$$R_e > \frac{200}{f^{1/2}}\left(\frac{4R}{k}\right) \tag{2-8}$$

where R_e is the Reynolds number. The actual Reynolds number of the flow condition is defined as:

$$R_e = \frac{4RV}{\nu} \tag{2-9}$$

where ν is the kinematic viscosity of the water. Resistance coefficients throughout the entire range of flow conditions can be obtained through the use of Plate 2-1.

c. Chezy Equation. The Chezy equation defines velocity in terms of the hydraulic radius, the slope S, and the Chezy resistance coefficient C in the form of

$$V = C(RS)^{1/2} \tag{2-10}$$

By equating S to h_f/L and rearranging terms in equation 2-10, the Chezy equation expresses the energy loss due to boundary roughness as

$$h_f = \frac{L}{R}\frac{V^2}{C^2} \tag{2-11}$$

The resistance coefficient, C, is dependent upon the Reynolds number and the effective roughness value. The C value can be determined through the use of Plate 2-1 or by equation 2-12:

$$C = 32.6\log\left(\frac{12.2R}{k}\right) \tag{2-12}$$

for fully turbulent flow conditions as defined by the relationship:

$$R_e > \frac{50\ CR}{k} \tag{2-13}$$

Chezy's C can also be determined through the use of the Darcy-Weisbach resistance coefficient, f , by equation 2-14:

$$C = \left(\frac{8g}{f}\right)^{1/2} \tag{2-14}$$

The characteristics of f in circular pipe flow have been thoroughly investigated by Nikuradse and Colebrook and White; however, a similar complete investigation of the characteristics of C in open channel flow have not been made due to the extra variables and wide range of surface roughness involved. However, reasonably accurate results should be obtained through the use of the Chezy equation.

d. Manning Equation. The Manning equation 2-15 defines velocity in terms of the hydraulic radius and slope, in a similar manner to the Chezy equation; however, the resistance coefficient is defined by the Manning's n value.

$$V = \frac{1.486\ R^{2/3}\ S^{1/2}}{n} \tag{2-15}$$

The constant 1.486 converts the metric equation to foot-second units. By equating $S = h_f/L$ and rearranging terms in equation 2-15, the Manning equation expresses the energy loss due to boundary roughness as

$$h_f = \frac{V^2 n^2\ L}{2.21\ R^{4/3}} \tag{2-16}$$

The Manning's resistance coefficient n , reported in numerous hydraulic publications, is founded on empiricism. It does not address the degree of turbulence or the interaction between the flow and boundary. The empiricism of this coefficient limits its accuracy when applied to conditions somewhat different from those from which it is derived. However, Manning's method is widely used due mainly to the large volume of reference data available to correlate resistance coefficients with boundary conditions and the ease in which the method can be used. When the design involves a significant amount of surface roughness energy loss resulting from fully turbulent flow, such as with a long spillway chute, the Manning's resistance coefficient may be calculated to account for the relative roughness effect by the use of

$$n = \frac{f^{1/2} R^{1/6}}{10.8} \tag{2-17}$$

or

$$n = 1.486 \frac{R^{1/6}}{C} \tag{2-18}$$

and the procedures described for equation 2-6 or 2-7. A review of energy loss computation using the Manning equation 2-16 modified to account for relative roughness by equations 2-6 or 2-7 and 2-17 or 2-18 will show that, if the effect of relative roughness is required, the Darcy-Weisbach or the Chezy method provides a more direct and simpler procedure.

e. Roughness Values. Values of effective roughness k normally are based on prototype measurements of flow over various boundary materials rather than physically measured values. Essentially all hydraulic textbooks provide extensive data of Chezy's C and Manning's n values; however, data are somewhat limited on effective roughness values k . Some suggested roughness values for various spillway surfaces are provided in the following tabulation:

Surface	Effective Roughness k, feet
Concrete	
For discharge design	0.007
For velocity design	0.002
Excavated rock	
Smooth and uniform	0.025-0.25
Jagged and irregular	0.15 -0.55
Natural vegetation	
Short grass	0.025-0.15
Long grass	0.10 -0.55
Scattered brush and weeds	0.15 -1.0

Due to the inability to predict the roughness that will be constructed, the designer should use maximum values in computing flow profiles and minimum values in computing energy losses required for terminal structure design.

2-11. Turbulent Boundary Layer Development Energy Loss. The surface roughness energy loss associated with free flow (ungated) on an overflow crest spillway with a P/H_d ratio greater than one is dependent upon the development of the turbulent boundary layer thickness. Boundary layer development is important to the designer because the principles of energy loss based upon the methods appropriate for fully turbulent flow are not valid until the boundary layer is fully developed. However, the use of the following procedure is dependent upon the spillway flow approach conditions conforming to the following assumptions:

a. The flow approaching the spillway must have potential flow velocity distribution (constant velocity throughout the flow depth).

b. The flow depth is large so that the depth of approach flow can be considered constant.

c. No restrictions exist at the spillway entrance that would cause an abrupt disturbance of the water and velocity distribution.

The turbulent boundary layer thickness δ (all values in feet) is a function of the length, L , along the spillway from the start of the crest curve and the effective roughness, k , described as

$$\frac{\delta}{L} = 0.08\left(\frac{L}{k}\right)^{-0.233} \tag{2.19}$$

The decrease in energy flux in the turbulent boundary layer caused by friction is by definition the energy thickness δ_3 . The decrease in the volume of flow in the boundary layer caused by friction is by definition the displacement thickness δ_1 . Based on experimental data the relationship between the displacement thickness δ_1 , the energy thickness δ_3 , and the turbulent boundary layer thickness is:

$$\delta_1 = 0.18\delta \tag{2-20}$$

$$\delta_3 = 0.22\delta \tag{2-21}$$

The potential flow velocity at any location T investigated on the spillway is determined from equation 2-22, using a trial procedure

$$h_T = d_P \cos\theta + \frac{u^2}{2g} \tag{2-22}$$

where

h_T = reservoir elevation minus spillway elevation at location T , feet
d_p = potential flow depth at location T , feet
θ = interior angle between spillway face at location T and horizontal, degrees
u = potential flow velocity, ft/sec

The spillway energy loss, H_L , in terms of feet of head, is defined by the following equation:

$$H_L = \frac{\delta_3 u^3}{2gq} \tag{2-23}$$

where q is the unit discharge in cubic feet per second per foot (ft^3 /sec/ft). The actual depth of flow, d , at the location under investigation is equal to the potential flow depth determined from equation 2-23 plus the displacement thickness from equation 2-20.

$$d = d_p + \delta_1 \tag{2-24}$$

The critical point is defined as the location where the turbulent boundary layer intersects the free surface flow, which is the location where the turbulent boundary layer thickness becomes equal to the actual flow depth. Downstream from the critical point, energy loss computations are based on fully turbulent flow, as discussed in paragraph 2-10, are appropriate. Reference is made to HDC Sheets and Charts 111-18 to 111-18/5 for additional information on procedures involved in determination of boundary layer development energy loss.

Section V. Hydraulic Jump Energy Dissipator

2-12. General.

a. Types of Energy Dissipators. Spillway energy dissipators are required to operate safely and effectively throughout a wide range of discharges, for extended periods of time, without having to shut down for emergency repairs. Energy dissipators normally used at CE dams are the hydraulic jump stilling basin, the roller bucket, and the flip bucket. Discussion on the selection and merits of each of these dissipators is presented in Chapter 7.

b. Unit Horsepower. When designing an energy dissipator, the horsepower per foot of width entering the dissipator should be determined. Unit horsepower, which provides an index of the severity of the entering energy conditions, can be expressed as

$$h_P = \frac{q\gamma(d_1 + h_e)}{550} \tag{2-25}$$

where

q = discharge per unit width of spillway, $ft^3/sec/ft$
γ = unit weight of water, lb/ft^3
d_1 = depth of flow at entrance to dissipator, feet
h_e = velocity head = $V_1^2/2g$ where V_1 = mean velocity of flow at entrance to dissipator, ft/sec

Plate 2-2 depicts the unit horsepower for a number of existing large spillways. This plate is presented to permit the designer to investigate operating experience with energy dissipators subjected to unit horsepower of a magnitude comparable to the projected design.

2-13. Hydraulic Jump Type Energy Dissipator. The hydraulic jump energy dissipator, defined as a stilling basin, is used to dissipate kinetic energy by the formation of a hydraulic jump. The hydraulic jump involves the principle of conservation of momentum. This principle states that the pressure plus momentum of the entering flow must equal the pressure plus momentum of the exiting flow plus the sum of the applied external forces in the basin. The hydraulic jump will form when the entering Froude number F_1, the entering flow depth d_1, and the sequent flow depth d_2 satisfy the following equation:

$$\frac{d_2}{d_1} = 0.5\left[\left(1 + 8F_1^2\right)^{1/2} - 1\right] \tag{2-26}$$

where

$$F_1 = \frac{V_1}{(gd_1)^{1/2}} \tag{2-27}$$

The energy losses within the basin and the forms of a hydraulic jump are dependent upon the entering Froude number. With Froude numbers F_1 less than 4.0, the jump is somewhat inefficient in energy dissipation and is hydraulically unstable. The entering flow oscillates between the bottom of the basin and the water surface, resulting in irregular period waves which will propagate downstream. EM 1110-2-1605 presents a discussion on the design of hydraulic jump stilling basins with entering Froude numbers less than 4.0. A well-stabilized and efficient jump will occur with Froude numbers F between 4.5 and 9.0. Jumps with Froude numbers F_1 greater than 9.0 are highly efficient in energy dissipation; however, a rough surface will exist that will propagate waves downstream. The energy loss ΔE resulting in a hydraulic jump is equal to the difference in specific energies before, E_1, and after, E_2, the jump which can be estimated by the following equation:

$$\Delta E = E_1 - E_2 = \frac{\left(d_2 - d_1\right)^3}{4d_1 d_2} \tag{2-28}$$

The length L_j of a hydraulic jump on a flat floor without baffles, end sill, or runout slope (not necessarily the stilling basin length) can be estimated by the following equations:

$$L_j = 8.0 d_1 F_1 \text{ for } F_1 > 5 \tag{2-29a}$$

$$L_j = 3.5 d_1 F_1^{1.5} \text{ for } 2 < F_1 < 5 \tag{2-29b}$$

The presence of baffles and/or end sills In the basin will shorten the jump length and reduce the d_2 depth required to produce the jump. The analysis of a hydraulic jump can be accomplished using the principle of conservation of momentum which requires that the rate of change of momentum entering and leaving the jump be equal to the summation of forces acting upon the fluid. The forces include the hydrostatic pressure force at each end of the jump which is expressed as

$$P_1 = \frac{\gamma d_1^2}{2} \tag{2-30}$$

$$P_3 = \frac{\gamma d_3^2}{2} \tag{2-31}$$

the force exerted by the baffles, expressed as

$$P_B = C_D \rho \left(\frac{V_B^2 \, h}{2} \right) \tag{2-32}$$

and the force exerted by the face of the end sill which is expressed as:

$$P_S = \gamma h_s \left(d_3 + \frac{h_s}{2} \right) \tag{2-33}$$

where

P_1 = hydrostatic pressure of the entering flow, lb/ft
P_3 = hydrostatic pressure of the exiting flow, lb/ft
d_3 = depth of flow above the end sill, feet
P_B = force exerted by the baffles, lb/ft
C_D = baffle drag coefficient
ρ = mass density of water pounds-seconds squared per feet to the fourth power ($lb\text{-}sec^2/ft^4$)
V_B = average velocity at face of the baffle, ft/sec
h = height of the baffle, feet
P_S = force exerted by the end sill, lb/ft
h_s = height of end sill, feet

Equation 2-33 assumes hydrostatic pressure distribution on the end sill. This assumption is considered valid unless the baffle piers are located too near the sill, resulting in a reduced pressure on the face of the end sill. The pressure reduction would require a theoretical increase in the downstream depth to provide the necessary force for jump stabilization. A friction force also exists along the basin wetted perimeter but is small enough to be neglected. Therefore, assuming two-dimensional flow, the momentum equation for a hydraulic jump which includes baffle piers and an end sill can be expressed as

$$\rho q V_1 - \rho q V_3 = P_3 - P_1 + P_B + P_S \tag{2-34}$$

where V_3 is the mean velocity at exit of dissipator or restated as

$$\gamma \left(\frac{q^2}{g d_1} - \frac{q^2}{g d_3} \right) = \gamma \left(\frac{d_3^2}{2} - \frac{d_1^2}{2} \right) + C_D \rho \left(\frac{V_B^2 \, h}{2} \right) + \gamma h_s \left(d_3 + \frac{h_s}{2} \right) \tag{2-35}$$

Solution of this equation for the required depth d_3 can be accomplished by successive trials for any specific baffle pier and end sill arrangement provided information is available to evaluate the baffle force. The baffle force is dependent upon the drag coefficient corresponding to the type of baffle used and the velocity in the vicinity of the baffle. The appropriate velocity can be estimated from Plate 2-3, which shows the distribution of velocity in a hydraulic jump. The baffle drag coefficient is a function of baffle shape and spacing. Limited information available on baffle drag coefficients indicates that the following values should be reasonable: 0.6 for a single row of baffles and 0.4 for a double row. Further discussion on baffles and end sills is found in Chapter 7.

2-14. Sidewall Dynamic Load. The turbulence created by the hydraulic jump imposes forces on the stilling basin sidewalls. The magnitude of the dynamic load is important in the structural design of the walls. Tests to determine sidewall forces were conducted at WES with an instrumented sidewall in a stilling basin that did not contain baffles or an end sill (item 19). These tests were conducted with Froude numbers F_1 that varied between 2.7 and 8.7, and resulted in the development of the following empirical equation:

$$R_m = 3.75 H_s^{-1.05} \rho V_1 q F_1^{-1.42} \tag{2.36}$$

where

R_m = average minimum static plus dynamic unit force at the toe of the hydraulic jump, lb/ft
H_s = spillway height, crest elevation minus stilling basin apron elevation, feet

The magnitude of the unit force on the sidewall varies along the length of the stilling basin. Plate 2-4 defines the variation in unit force by use of the normalizing functions, described by equations 2-37 through 2-39, versus the distance ratio x/L_b .

$$\frac{R - R_m}{R_s - R_m} = C \tag{2-37}$$

$$\frac{(R_+) - R_m}{R_s - R_m} = C_+ \tag{2-38}$$

$$\frac{(R_-) - R_m}{R_s - R_m} = C_- \tag{2-39}$$

where

x = distance measured from the point of intersection of the spillway slope and the basin apron to the center line of the wall unit being analyzed
L_b = length of the stilling basin, feet
R, R_+, R_- = average unit resultant force, maximum instantaneous unit resultant force, and minimum instantaneous unit resultant force, respectively, acting on the sidewall when the actual

depth of tailwater d_{TW} is less than or equal to d2 or the basin wall height, lb/ft

R_S = static unit force on the sidewall unit due to the theoretical sequent depth for a hydraulic jump, lb/ft

When $d_{TW} > d_2$, R, R_+, and R_- must be adjusted as shown by equation 2-40 through 2-42 to account for the increased force resulting from the difference between d_{TW} and d_2:

$$R_a = R + \frac{\gamma}{2}\left(d_{TW}^2 - d_2^2\right) \tag{2-40}$$

$$R_{a_+} = R_+ + \frac{\gamma}{2}\left(d_{TW}^2 - d_2^2\right) \tag{2-41}$$

$$R_{a_-} = R_- + \frac{\gamma}{2}\left(d_{TW}^2 - d_2^2\right) \tag{2-42}$$

where R_a, R_{a_+}, and R_{a_-} are the adjusted average unit resultant force, the adjusted maximum instantaneous unit resultant force, and the adjusted minimum unit resultant force, respectively. The distance above the stilling basin apron, Y, to the resultant of the unit force acting on the basin wall is determined by the use of Plate 2-5, which defines the relationship between Y/d_{TW} and X/L_b. Appendix E includes an example problem illustrating the recommended application for estimating the magnitude and locations of the resultant dynamic forces acting on a stilling basin sidewall.

Section VI. Cavitation

2-15. General. Cavitation is defined as the formation of a gas and water vapor phase within a liquid resulting from excessively low localized pressures. When associated with the design of spillways, cavitation is important because the vaporization occurs on or near the nonfluid boundary (spillway surface) resulting from localized boundary shape conditions. Cavitation damage results when the gas and water vapor-filled void is swept from the low-pressure zone into an adjacent higher pressure zone which will not support cavitation, causing the void to collapse. The collapse of the void results in extremely high pressures, and when they occur at or near the nonfluid boundary, will form a small pit. When given sufficient time, numerous void collapses result in numerous small pits which eventually overlap, leading to larger holes. This damage, in turn, aggravates the localized low-pressure zone, thereby creating a self-breeding continuation of the damage. The existence and extent of cavitation damage are dependent upon the boundary shape, the damage resistance characteristics of the boundary, the flow velocity, the flow depth, the elevation of the structure above sea level, and the length of time the cavitation occurs. Cavitation damage can be detected at one or more locations in essentially all high-velocity flow structures; however, and fortunately, most damage is minor and results from cavitation conditions at or very near the incipient damage level. When incipient levels are exceeded, serious damage will occur. At Libby Dam, a construction-related misalignment of the parabolic-shaped invert of the open channel flow sluices resulted in cavitation damage that removed concrete and reinforcing steel throughout an area 54 feet in length, up to 7 feet wide, and up to 2.5 feet deep on both the floor and the right wall (item 47). At Hoover, Yellowtail, and Glen Canyon Dams, severe cavitation damage occurred in tunnel spillways near the tangent point of the vertical curve which decreases the slope of the spillway. The spillways at these dams are tunnel-type structures which were operating at open channel flow conditions with average flow velocities in excess of 100 ft/sec when the damage occurred. Similar flow conditions can exist on a spillway chute. Damage to concrete surfaces can occur at velocities significantly less than 100 ft/sec provided the correct combination of cavitation parameters exists. As a rule of thumb, cavitation should be investigated whenever flow velocities are in excess of 35 ft/sec.

2-16. Cavitation Damage. The damage potential resulting from cavitation is dependent upon the boundary shape, the damage resistance characteristics of the boundary, the flow velocity and depth, the elevation of the structure above mean sea level, and the length of time cavitation occurs. The boundary shape, velocity, and elevation are related by the cavitation index, σ, which is derived from the energy equation:

$$\frac{V_0^2}{2g} + \frac{P_0}{\gamma} + Z_0 = \frac{V_1^2}{2g} + \frac{P_1}{\gamma} + Z_1 \tag{2-43}$$

where P is the absolute pressure, lb/ft^2. With $H = P/\gamma$ the comparable equation is

$$\frac{H_1 - H_0}{\frac{V_0^2}{2g}} = 1 - \left(\frac{V_1}{V_0}\right)^2 + \frac{Z_0 - Z_1}{\frac{V_0^2}{2g}} \tag{2-44}$$

For high velocities the elevation term in equation 2-44 can be ignored. The dimensionless parameter on the left side of the equation is known as the pressure parameter. Replacing H_1 with the absolute head required for vaporization of water at the elevation of the structure above sea level and rearranging terms in order that σ will be positive, the flow cavitation index is

$$\sigma = \frac{H_0 - H_V}{\frac{V_0^2}{2g}} \tag{2-45}$$

where

H_0 = reference head, feet
H_V = vapor head of water, feet

Various investigators have experimentally determined the σ-incipient cavitation relationship for a number of specific boundary shapes. These experimentally derived data have been reduced to curves describing the incipient cavitation level for specific boundary shapes (Plates 2-6 through 2-9). Cavitation damage can be expected if a specific u-boundary shape relationship can be plotted on or to the right side of the curve. When this condition is evident, **a design change must be made that either increases the σ value,** smoothes the boundary shape, or both. **As σ decreases below the incipient** cavitation level, the cavitation damage potential increases very rapidly. Investigations (item 14) have found that the cavitation energy absorbed by the nonfluid boundary increases by the eleventh power of the velocity.

2-17. Cavitation Damage Prevention. Cavitation-induced damage can be prevented by a number of methods. As shown in paragraph 2-16, damage can be prevented by increasing the cavitation index and/or by providing a smoother boundary shape. However, changes of this type are usually impractical or at best difficult to accomplish due to the physical limitations imposed by the required design and construction practices. Changing the damage resistance characteristics of the boundary will inhibit the damage produced over-a finite period of time. As an example, structural concrete exposed to cavitation resulting from a flow velocity of 98 ft/sec for 3 hours resulted in a hole 0.5 inch deep. Under the same conditions with polymerized concrete, the same size hole resulted after 6,000 hours. The use of hardened boundaries also has physical limitations, and results only in resisting the cavitation forces for a given period of time. A relatively new and very effective method of preventing cavitation-induced damage is to disperse a quantity of air along the flow boundary. This is achieved by passing the water over an aeration slot specially designed to entrain air along the boundary. This method has been used to prevent cavitation damage at various high-velocity flow facilities including Libby Dam sluices (item 46). Prototype tests of boundary pressures were obtained at identical locations and hydraulic conditions for pre- and post-boundary aeration. These tests showed that aeration of the boundary resulted in raising instantaneous pressures that were very close to absolute zero to pressures near atmospheric. Data collected from these tests were used to derive the cavitation index. The post-boundary aeration cavitation index showed an average increase of about 50 percent above the preaeration condition. The aeration slot geometry and location must be designed for the specific application. Some design guidance has been developed (item 13) to assist in aeration slot design and should be used to develop an initial design. Until significantly more experience, data, and design guidance are developed, model studies of aeration slot design are recommended.